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Marian Apostol ^a , Victor Barsan ^a & Constantin Nantea ^a

^a Central Instirute of Physics, Magurele-Bucharest, NG-6, Romania

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ON THE CUT-OFF PARAMETERS IN THE BOSONIZATION TECHNIQUE

MARIAN APOSTOL, VICTOR BARSAN AND CONSTANTIN MANTEA Central Institute of Physics, Magurele-Bucharest, MG-6, Romania

Abstract Jordan's boson representation and cut-off regularization for the one-dimensional two-fermion model (TFT) is used to get the equivalence of the model with the two-dimensional Coulomb gas and sine-Gordon model. The scaling equations for the coupling constants are thereby obtained up to the third order.

POINT-SPLITTING REGULARIZATION

The Tomonaga-Luttinger model extended to include the backscattering and umklapp scattering interactions (TFM) turns out to be an exactly soluble model for particular values of the coupling constants within the bosonization approach. 3,4 However the cut-off parameter & appearing in the boson represen tation 3 induces an unphysical dependence of the fermion gap $(\sim \propto^{-1}, \propto \rightarrow 0)$ of the TFU and an unsatisfactory behaviour of the solutions of the scaling equations for the coupling constants. 2 Moreover, the correlation functions of the Tomonaga-Luttinger model calculated by means of the boson representation do not coincide with those obtained by direct diagram summation. 5 It seems that the cut-off parameter of is too strong as it leaves no room for making the dissociation between the bandwidth and momentum transfer cut-off. In addition, the unknown nature of this cut-off parameter makes difficult the comparison between the charge-density correlation functions of the TFT and Fermi gas model ($g_{1u,1}$ contributions).

It has been shown that the full form of the boson representation and the cut-off regularization have been given many years ago by Jordan. When generalized to the TFT Jordan's boson representation and point-splitting regularization remove the aforementioned inconsistencies. In particular, the cut-off parameter appearing in the point-splitting regularization turns out to corresponds to a bandwidth cut-off, the momentum transfer cut-off parameter r being introduced into the TFT in order to ensure the finiteness of the interaction contributions. When applied to the TFT the Jordan bosonization approach should be operated by keeping r finite and letting contributions of the Fermi sea have been subtracted. Doing so one obtains the fermion gap of the TFM proportional to r-1 and not to cut-1, having thus a finite value.

SCALING EQUATIONS

Taking use of the α^{-1} -dependence of the gap and the equivalence between the TFT and the two-dimensional electron gas a specific scale function has been proposed for the coupling constant $g_{1\perp}$ (backscattering strength). Kosterlitz-Thouless scaling equations for $g_{1\parallel}$ and $g_{1\perp}$ have been afterwards generalized to the next-to-leading order (third order) by assuming that they must acquire the same form for $g_{1\parallel}=g_{1\perp}$. However, although the equivalence with the two-dimensional Coulomb gas still holds, the coupling constant $g_{1\perp}$ scales with r when the Jordan approach is used and one can not use any longer the aforementioned procedure. Instead, the equivalence of the TFT with the sine-Gordon model can be used together with the scaling approach to the latter. The aforementioned equivalence is straightforwardly obtained by using Jordan approach and the flow equations result at once:

$$dg_{1N}/d\varepsilon = -g_{1L}^{2}(1 - \frac{1}{2}g_{1N}),$$

$$dg_{1L}/d\varepsilon = -g_{1L}[g_{1N} + \frac{1}{2}(g_{1N}^{2} + 3g_{1L}^{2})],$$
(1)

where & = lnr is the flow parameter. The corresponding invariant combination

$$g_{1}^{2} - g_{1}^{2} - (3/2)g_{1}^{2}g_{11} + \frac{1}{2}g_{11}^{3} = \text{const.}$$
 (2)

only slightly differs (instead of the 3/2 coefficient appears 1) from that previously given. 2,8 This difference can be traced back to the asymmetrical form of Eqs.(1) which do not coincide for $g_{11} = g_{11}$. Assuming the coincidence the coefficient 3/4 in Eqs.(1) changes the sign. However the coincidence of the flow equations for $\varepsilon_{1\,\mathrm{N}}$ = $\varepsilon_{1\,\mathrm{L}}$ is a reasonable assumption when one deals with the Fermi gas model⁵ where the two contributions brought by g1% and g11 terms have the same mathematical structure : products of four fermion field operators. In the bosonized version of the Fermi gas model (TFM) the gam contribution includes products of two boson operators while the g_{11} contribution is an integral over products of four exponential functions having boson operators in the expression of their exponents (boson representation). It seems that such different mathematical structures induce different (asymmetrical) scaling equations for g_{11} and g_{1} . That is even if at some point ε_0 we have $g_{1k}(\varepsilon_0) = g_{1k}(\varepsilon_0)$ the invariant given by Eq.(2) does not conserve this equality and thus we have no symmetry trajectory $g_{1k}(\xi) = g_{1k}(\xi)$.

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