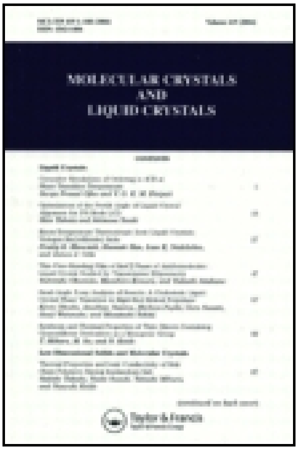
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On The Cut-off Parameters in the Bosonization Technique

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ON THE CUT-OFF PARAMETERS IN THE BOSONIZATION TECHNIQUE

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Abstract Jordan's boson representation and cut-off regularization for the one-dimensional two-fermion model (TFM) is used to get the equivalence of the model with the two-dimensional Coulomb gas and sine-Gordon model. The scaling equations for the coupling constants are thereby obtained up to the third order.

POINT-SPLITTING REGULARIZATION

The Tomonaga-Luttinger model extended to include the backscattering[1] and umklapp scattering[2] interactions (TFM) turns out to be an exactly soluble model for particular values of the coupling constants within the bosonization approach.[3,4] However the cut-off parameter α appearing in the boson representation[3] induces an unphysical dependence of the fermion gap ($\sim \alpha^{-1}, \alpha \to 0$) of the TFM and an unsatisfactory behaviour of the solutions of the scaling equations for the coupling constants.[2] Moreover, the correlation functions of the Tomonaga-Luttinger model calculated by means of the boson representation do not coincide with those obtained by direct diagram summation.[5] It seems that the cut-off parameter α is too strong as it leaves no room for making the dissociation between the bandwidth and momentum transfer cut-off. In addition, the unknown nature of this cut-off parameter makes difficult the comparison between the charge-density correlation functions of the TFM and Fermi gas model ($g_{1\parallel,\perp}$ contributions).[5]

It has been shown[6] that the full form of the boson representation and the cut-off regularization have been given many years ago by Jordan.[7] When generalized to the TFM Jordan's boson representation and point-splitting regularization[6] remove the aforementioned inconsistencies. In particular, the α cut-off parameter appearing in the point-splitting regularization turns out to corresponds to a bandwidth cut-off, the momentum transfer cut-off parameter r being introduced into the TFM in order to ensure the finiteness of the interaction contributions. When applied to the TFM the Jordan bosonization approach should be operated by keeping r finite and letting α go to zero everywhere the divergencies brought by the infinite filling of the Fermi sea have been subtracted. Doing so one obtains the fermion gap of the TFM proportional to r^{-1} and not to α^{-1}, having thus a finite value.

SCALING EQUATIONS

Making use of the α^{-1}-dependence of the gap and the equivalence between the TFM and the two-dimensional electron gas a specific scale function has been proposed[2] for the coupling constant $g_{1\perp}$ (backscattering strength). Kosterlitz-Thouless scaling equations for $g_{1\parallel}$ and $g_{1\perp}$ have been afterwards generalized to the next-to-leading order (third order) by assuming that they must acquire the same form for $g_{1\parallel} = g_{1\perp}$.[2,8] However, although the equivalence with the two-dimensional Coulomb gas still holds, the coupling constant $g_{1\perp}$ scales with r when the Jordan approach is used and one can not use any longer the aforementioned procedure. Instead, the equivalence of the TFM with the sine-Gordon model can be used together with the scaling approach to the latter.[9] The aforementioned equivalence is straightforwardly obtained by using Jordan approach and the flow equations result at once :

$$dg_{1\parallel}/d\varepsilon = -g_{1\perp}^2(1 - \tfrac{1}{2}g_{1\parallel}),$$
$$dg_{1\perp}/d\varepsilon = -g_{1\perp}\left[g_{1\parallel} + \tfrac{1}{4}(g_{1\parallel}^2 + 3g_{1\perp}^2)\right], \qquad (1)$$

where $\varepsilon = \ln r$ is the flow parameter. The corresponding invariant combination

$$g_{1\perp}^2 - g_{1\parallel}^2 - (3/2)g_{1\perp}^2 g_{1\parallel} + \tfrac{1}{2}g_{1\parallel}^3 = \text{const.} \qquad (2)$$

only slightly differs (instead of the 3/2 coefficient appears $\frac{1}{2}$) from that previously given.[2,8] This difference can be traced back to the asymmetrical form of Eqs.(1) which do not coincide for $g_{1\parallel} = g_{1\perp}$. Assuming the coincidence the coefficient 3/4 in Eqs.(1) changes the sign. However the coincidence of the flow equations for $g_{1\parallel} = g_{1\perp}$ is a reasonable assumption when one deals with the Fermi gas model[5] where the two contributions brought by $g_{1\parallel}$ and $g_{1\perp}$ terms have the same mathematical structure : products of four fermion field operators. In the bosonized version of the Fermi gas model (TFM) the $g_{1\parallel}$ contribution includes products of two boson operators while the $g_{1\perp}$ contribution is an integral over products of four exponential functions having boson operators in the expression of their exponents (boson representation). It seems that such different mathematical structures induce different (asymmetrical) scaling equations for $g_{1\parallel}$ and $g_{1\perp}$. That is even if at some point ε_0 we have $g_{1\parallel}(\varepsilon_0) = g_{1\perp}(\varepsilon_0)$ the invariant given by Eq.(2) does not conserve this equality and thus we have no symmetry trajectory $g_{1\parallel}(\varepsilon) = g_{1\perp}(\varepsilon)$.

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